Natural Language Processing with Deep Learning

CS224N/Ling284

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Lecture 2: Word Vectors

Lecture Plan

1. Word meaning (15 mins)
2. Word2vec introduction (20 mins)
3. Research highlight (Danqi) (5 mins)
4. Word2vec objective function gradients (25 mins)
5. Optimization refresher (10 mins)

Fire alarm allowance: 5 mins

1. How do we represent the meaning of a word?

Definition: **meaning** (Webster dictionary)

- the idea that is represented by a word, phrase, etc.
- the idea that a person wants to express by using words, signs, etc.
- the idea that is expressed in a work of writing, art, etc.

Commonest linguistic way of thinking of meaning:

- signifier $\Leftrightarrow$ signified (idea or thing) = denotation

How do we have usable meaning in a computer?

Common answer: Use a taxonomy like WordNet that has hypernyms (is-a) relationships and synonym sets (here, for *good*):

```python
from nltk.corpus import wordnet as wn
panda = wn.synset('panda.n.01')
hyper = lambda s: s.hypernyms()
list(panda.closure(hyper))
```

```
[Synset('procyonid.n.01'),
Synset('carnivore.n.01'),
Synset('placental.n.01'),
Synset('mammal.n.01'),
Synset('vertebrate.n.01'),
Synset('chordate.n.01'),
Synset('animal.n.01'),
Synset('organism.n.01'),
Synset('living_thing.n.01'),
Synset('whole.n.02'),
Synset('object.n.01'),
Synset('physical_entity.n.01'),
Synset('entity.n.01')]
```

S: (adj) full, good
S: (adj) estimable, good, honorable, respectable
S: (adj) beneficial, good
S: (adj) good, just, upright
S: (adj) adept, expert, good, practiced, proficient, skillful
S: (adj) dear, good, near
S: (adj) good, right, ripe
...
S: (adv) well, good
S: (adv) thoroughly, soundly, good
S: (n) good, goodness
S: (n) commodity, trade good, good

Problems with this discrete representation

- Great as a resource but missing nuances, e.g., **synonyms**:
  - adept, expert, good, practiced, proficient, skillful?
- Missing new words (impossible to keep up to date): wicked, badass, nifty, crack, ace, wizard, genius, ninja
- Subjective
- Requires human labor to create and adapt
- Hard to compute accurate word similarity →

Problems with this discrete representation

The vast majority of rule-based **and** statistical NLP work regards words as atomic symbols: hotel, conference, walk

In vector space terms, this is a vector with one 1 and a lot of zeroes

$$[0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]$$

Dimensionality: 20K (speech) – 50K (PTB) – 500K (big vocab) – 13M (Google 1T)

We call this a "one-hot" representation

It is a **localist** representation

From symbolic to distributed representations

Its problem, e.g., for web search

- If user searches for [Dell notebook battery size], we would like to match documents with "Dell laptop battery capacity"
- If user searches for [Seattle motel], we would like to match documents containing "Seattle hotel"

But

$$\begin{array}{l} \text{motel } [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]^T \\ \text{hotel } [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] = 0 \end{array}$$

Our query and document vectors are **orthogonal**

There is no natural notion of similarity in a set of one-hot vectors

Could deal with similarity separately;

instead we explore a direct approach where vectors encode it

Distributional similarity based representations

You can get a lot of value by representing a word by means of its neighbors

"You shall know a word by the company it keeps"

(J. R. Firth 1957: 11)

One of the most successful ideas of modern statistical NLP

government debt problems turning into banking crises as has happened in

saying that Europe needs unified banking regulation to replace the hodgepodge

↖ These words will represent *banking* ↗

Word meaning is defined in terms of vectors

We will build a dense vector for each word type, chosen so that it is good at predicting other words appearing in its context

… those other words also being represented by vectors … it all gets a bit recursive

$$\textit{linguistics} = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 0.109 \\ -0.542 \\ 0.349 \\ 0.271 \end{pmatrix}$$

Basic idea of learning neural network word embeddings

We define a model that aims to predict between a center word w_t and context words in terms of word vectors

$$p(\text{context} \mid w_t) = \ldots$$

which has a loss function, e.g.,

$$J = 1 - p(w_{-t} \mid w_t)$$

We look at many positions t in a big language corpus

We keep adjusting the vector representations of words to minimize this loss

Directly learning low-dimensional word vectors

Old idea. Relevant for this lecture & deep learning:

- Learning representations by back-propagating errors (Rumelhart et al., 1986)
- **A neural probabilistic language model** (Bengio et al., 2003)
- NLP (almost) from Scratch (Collobert & Weston, 2008)
- A recent, even simpler and faster model: word2vec (Mikolov et al. 2013) → intro now

2. Main idea of word2vec

Predict between every word and its context words!

Two algorithms

1. **Skip-grams (SG)**

   Predict context words given target (position independent)

2. Continuous Bag of Words (CBOW)

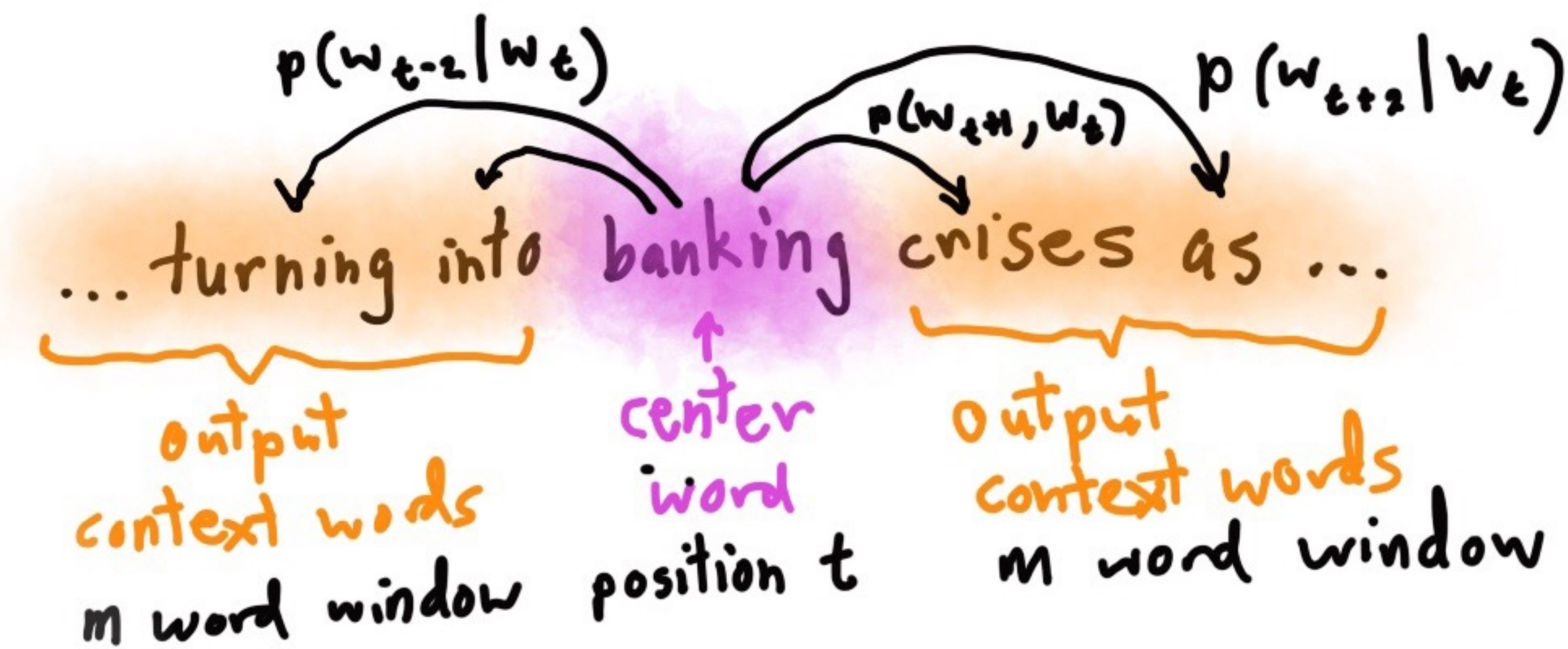
   Predict target word from bag-of-words context

Two (moderately efficient) training methods

1. Hierarchical softmax
2. Negative sampling

Naïve softmax

Skip-gram prediction



Details of word2vec

For each word $t = 1 \ldots T$, predict surrounding words in a window of "radius" m of every word.

Objective function: Maximize the probability of any context word given the current center word:

$$J'(\theta) = \prod_{t=1}^{T} \prod_{\substack{-m \le j \le m \\ j \ne 0}} p(w_{t+j} \mid w_t ; \theta)$$

Negative Log Likelihood

$$J(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-m \le j \le m \\ j \ne 0}} \log p(w_{t+j} \mid w_t)$$

Where θ represents all variables we will optimize

The objective function – details

- Terminology: Loss function = cost function = objective function
- Usual loss for probability distribution: Cross-entropy loss
- With one-hot w_{t+j} target, the only term left is the negative log probability of the true class
- More on this later…

Details of Word2Vec

Predict surrounding words in a window of radius *m* of every word

For $p(w_{t+j}|w_t)$ the simplest first formulation is

$$p(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)}$$

where *o* is the outside (or output) word index, *c* is the center word index, v_c and u_o are "center" and "outside" vectors of indices *c* and *o*

Softmax using word *c* to obtain probability of word *o*

Dot products

Dot product

$$u^T v = u.v = \sum_{i=1}^{n} u_i v_i$$

Bigger if u and v are more similar!

Iterate over $w=1 \dots W$: $u_w^T v$ means:

Work out how similar each word is to v !

$$p(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)}$$

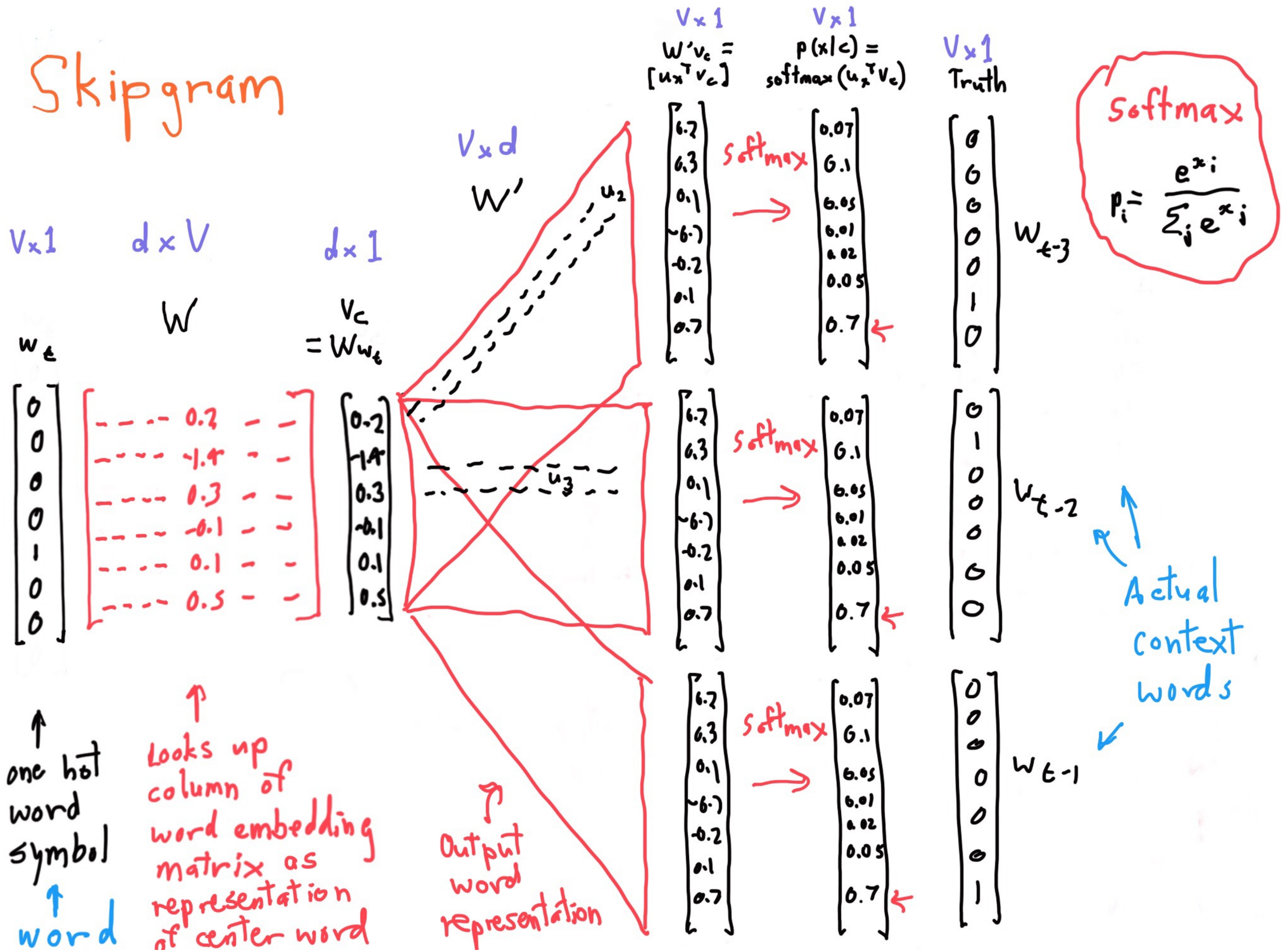
Softmax function: Standard map from $\mathbb{R}^V$ to a probability distribution

Softmax

Exponentiate to make positive

$$p_i = \frac{e^{u_i}}{\sum_j e^{u_j}}$$

Normalize to give probability



Skipgram
V x 1
d x V
d x 1
V x d
W
W'
w_t
$v_c = W w_t$
u_2
u_3
$W' v_c = [u_x^T v_c]$
$p(x|c) = \mathrm{softmax}(u_x^T v_c)$
Truth
softmax
w_{t-3}
w_{t-2}
w_{t-1}
Softmax
$p_i = \frac{e^{x_i}}{\sum_j e^{x_j}}$
Actual context words
one hot word symbol
word
Looks up column of word embedding matrix as representation of center word
Output word representation

To train the model: Compute all vector gradients!

- We often define the set of all parameters in a model in terms of one long vector θ
- In our case with *d*-dimensional vector and *V* many words:
- We then optimize these parameters

$$\theta = \begin{bmatrix} v_{aardvark} \\ v_a \\ \vdots \\ v_{zebra} \\ u_{aardvark} \\ u_a \\ \vdots \\ u_{zebra} \end{bmatrix} \in \mathbb{R}^{2dV}$$

Note: Every word has two vectors! Makes it simpler!

4. Derivations of gradient

- Whiteboard – see video if you're not in class ;)
- The basic Lego piece
- Useful basics: $\frac{\partial \mathbf{x}^T \mathbf{a}}{\partial \mathbf{x}} = \frac{\partial \mathbf{a}^T \mathbf{x}}{\partial \mathbf{x}} = \mathbf{a}$
- If in doubt: write out with indices

- Chain rule! If $y = f(u)$ and $u = g(x)$, i.e. $y = f(g(x))$, then:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Chain Rule

- Chain rule! If *y* = *f*(*u*) and *u* = *g*(*x*), i.e. *y* = *f*(*g*(x)), then:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{df(u)}{du}\frac{dg(x)}{dx}$$

- Simple example: $\frac{dy}{dx} = \frac{d}{dx}5(x^3+7)^4$

$$y = f(u) = 5u^4 \qquad u = g(x) = x^3 + 7$$

$$\frac{dy}{du} = 20u^3 \qquad \frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = 20(x^3+7)^3 . 3x^2$$

Interactive Whiteboard Session!

$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} \sum_{-m \leq j \leq m, j \neq 0} \log p(w_{t+j} | w_t)$$

Let's derive gradient for center word together
For one example window and one example outside word:

$$\log p(o|c) = \log \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)}$$

You then also also need the gradient for context words (it's similar; left for homework). That's all of the paramets θ here.

Objective Function

Maximize $J'(\theta) = \prod_{t=1}^{T} \prod_{\substack{-m \le j \le m \\ j \ne 0}} p(w_{t+j} \mid w_t; \theta)$

Or minimize neg. log likelihood $J(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-m \le j \le m \\ j \ne 0}} \log p(w_{t+j} \mid w_t)$

[negate to minimize; log is monotone]

Text length

window size

where

$$p(o \mid c) = \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)}$$

word IDs

Each word type (vocab entry) has <u>two</u> word representations: as center word and context word

We now take derivatives to work out minimum

$$\frac{\partial}{\partial v_c} \log \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w v_c)}$$

$$= \underbrace{\frac{\partial}{\partial v_c} \log \exp(u_o^T v_c)}_{1} - \underbrace{\frac{\partial}{\partial v_c} \log \sum_{w=1}^{V} \exp(u_w^T v_c)}_{2}$$

1 $$\frac{\partial}{\partial v_c} \underbrace{\log \exp}_{\text{inverses}} (u_o^T v_c) = \frac{\partial}{\partial v_c} u_o^T v_c = u_o$$

Vector!

Not high school single variable calculus

You can do things one variable at a time, and this may be helpful when things get gnarly.

$$\forall j \quad \frac{\partial}{\partial (v_c)_j} u_o^T v_c = \frac{\partial}{\partial (v_c)_j} \sum_{i=1}^{d} (u_o)_i (v_c)_i$$

$$= (u_o)_j$$

Each term is zero except when $i=j$

② $$\frac{\partial}{\partial v_c} \underbrace{\log}_{f} \underbrace{\sum_{w=1}^{V} \exp(u_w^T v_c)}_{z = g(v_c)}$$

$$= \underbrace{\frac{1}{\sum_{w=1}^{V} \exp(u_w^T v_c)}}_{} \cdot \underbrace{\frac{\partial}{\partial v_c} \sum_{x=1}^{V} \exp(u_x^T v_c)}_{\frac{\partial z}{\partial v_c}}$$

Important to change index

$$\frac{\partial}{\partial v_c} f(\overbrace{g(v_c)}^{z}) = \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial v_c}$$

Use chain rule

$$= \frac{1}{\sum_{w=1}^{V} \exp(u_w^T v_c)} \cdot \left(\sum_{x=1}^{V} \frac{\partial}{\partial v_c} \underbrace{\exp}_{f} (\underbrace{u_x^T v_c}_{z = g(v_c)}) \right)$$

Move deriv inside sum

$$\left(\sum_{x=1}^{V} \exp(u_x^T v_c) \frac{\partial}{\partial v_c} u_x^T v_c \right)$$

Chain rule

$$\left(\sum_{x=1}^{V} \exp(u_x^T v_c) \, u_x \right)$$

$$\frac{\partial}{\partial v_c} \log(p(o|c)) = u_o - \frac{1}{\sum_{w=1}^{V} \exp(u_w^T v_c)} \cdot \left(\sum_{x=1}^{V} \exp(u_x^T v_c) u_x \right)$$

$$= u_o - \sum_{x=1}^{V} \frac{\exp(u_x^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)} u_x$$

Distribute term across sum

$$= u_o - \underbrace{\sum_{x=1}^{V} p(x|c)\, u_x}$$

This is an expectation: average over all context vectors weighted by their probability

= observed − expected

This is just the derivatives for the center vector parameters
Also need derivatives for output vector parameters
(they're similar)
Then we have derivative w.r.t. all parameters and can minimize

Calculating all gradients!

- We went through gradient for each center vector v in a window
- We also need gradients for outside vectors u
- Derive at home!

- Generally in each window we will compute updates for all parameters that are being used in that window.
- For example, window size m = 1, sentence:

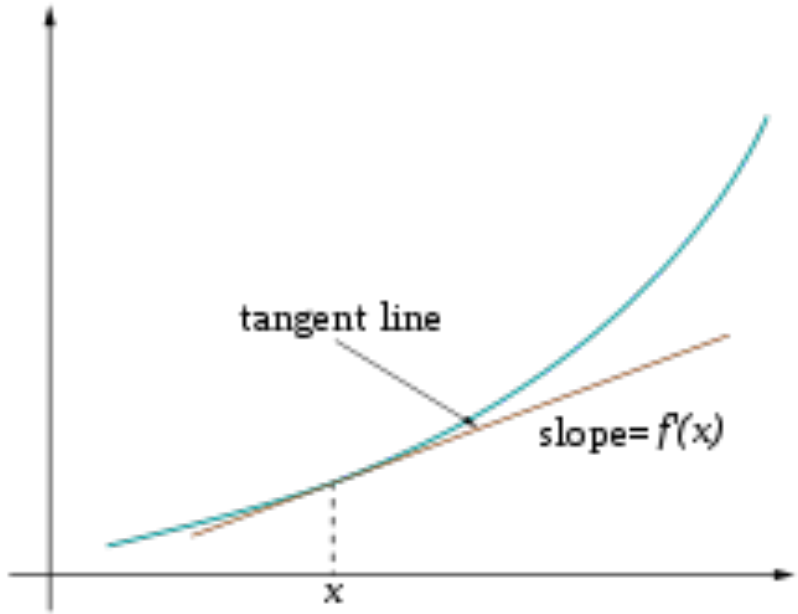
  "We like learning a lot"
- First window computes gradients for:
  - internal vector v_{like} and external vectors u_{We} and u_{learning}
- Next window in that sentence?

5. Cost/Objective functions

We will optimize (maximize or minimize) our objective/cost functions

For now: minimize → gradient descent

Trivial example: (from Wikipedia)
Find a local minimum of the function
$f(x) = x^4 - 3x^3 + 2$, with derivative $f'(x) = 4x^3 - 9x^2$




```
x_old = 0
x_new = 6 # The algorithm starts at x=6
eps = 0.01 # step size
precision = 0.00001

def f_derivative(x):
    return 4 * x**3 - 9 * x**2

while abs(x_new - x_old) > precision:
    x_old = x_new
    x_new = x_old - eps * f_derivative(x_old)

print("Local minimum occurs at", x_new)
```

Subtracting a fraction of the gradient moves you towards the minimum!

Gradient Descent

- To minimize $J(\theta)$ over the full batch (the entire training data) would require us to compute gradients for all windows

- Updates would be for each element of θ :

$$\theta_j^{new} = \theta_j^{old} - \alpha \frac{\partial}{\partial \theta_j^{old}} J(\theta)$$

- With step size α
- In matrix notation for all parameters:

$$\theta^{new} = \theta^{old} - \alpha \frac{\partial}{\partial \theta^{old}} J(\theta)$$

$$\theta^{new} = \theta^{old} - \alpha \nabla_{\theta} J(\theta)$$

Vanilla Gradient Descent Code

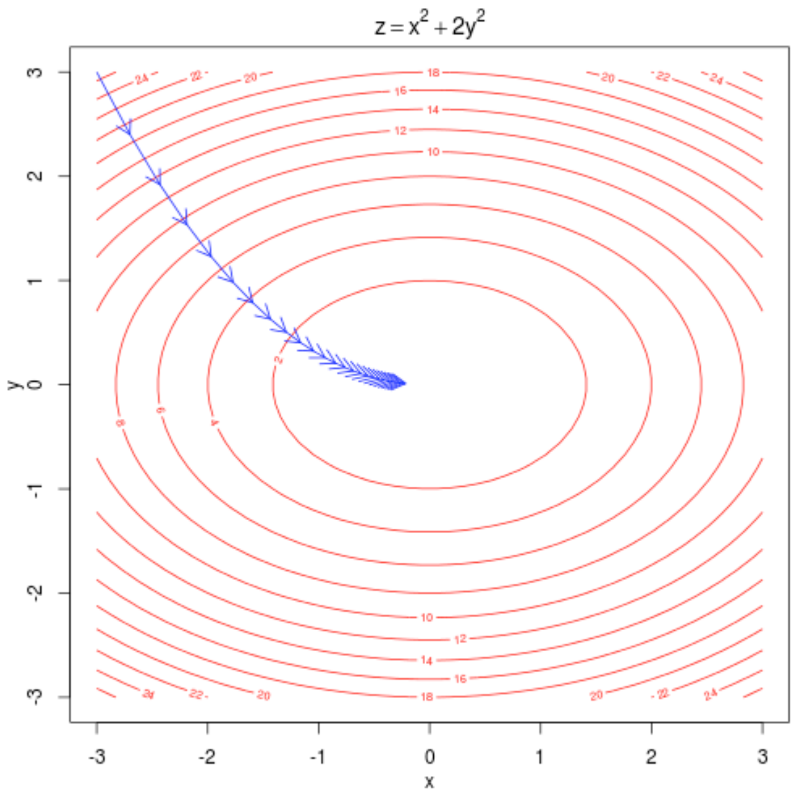
$$\theta^{new} = \theta^{old} - \alpha \nabla_{\theta} J(\theta)$$

```
while True:
    theta_grad = evaluate_gradient(J,corpus,theta)
    theta = theta - alpha * theta_grad
```

Intuition

For a simple convex function over two parameters.

Contour lines show levels of objective function



Stochastic Gradient Descent

- But Corpus may have 40B tokens and windows
- You would wait a very long time before making a single update!

- Very bad idea for pretty much all neural nets!
- Instead: We will update parameters after each window t
  → Stochastic gradient descent (SGD)

$$\theta^{new} = \theta^{old} - \alpha \nabla_\theta J_t(\theta)$$

```
while True:
    window = sample_window(corpus)
    theta_grad = evaluate_gradient(J,window,theta)
    theta = theta - alpha * theta_grad
```